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Mach's Principle and the Deceleration Parameter

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We relate the deceleration parameter q to the mass and "radius" of the observable universe, and set a theoretical estimate for q by means of Mach's principle as represented by Whitrow and Randall relation $q \sim 1$.

Mach's principle states, generally speaking, that local inertia is determined by the overall distribution of mass in the whole universe. The part played by Mach's principle in the genesis of relativistic cosmology has been exhaustively discussed by Barbour (1990).

Tod (1994) gave evidence to support a formulation of the cosmological part of Mach's principle as the requirement that the initial singularity of space-time is isotropic, and suggested that this principle may become a theorem of quantum gravity.

Dicke (1964) determined, by employing Mach's principle, that a satisfactory gravitational potential could not be (a) a scalar potential in four-dimensional spacetime, (b) a vector potential in spacetime; however, it should be a second-order tensor potential, and in its simplest version it should, of course, be the metric tensor in a Riemannian geometry.

Whitrow (1946) and Whitrow and Randall (1951) and later Brans and Dicke (1961) concluded that by equating the inertial cosmic energy

$$Mc^2 \tag{1}$$

to the self-gravitational energy

$$\frac{3}{5}\frac{GM^2}{R'} \tag{2}$$

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for a sufficiently large sphere of mass M and radius R', Mach's principle should be fulfilled, thus obtaining the Whitrow–Randall–Sciama–Brans–Dicke relation

$$\frac{GM}{c^2 R'} \sim 1 \tag{3}$$

The deceleration parameter in cosmology is defined by

$$q = -\frac{\ddot{R}R}{\dot{R}^2} \tag{4}$$

where R is the scale factor in the Robertson–Walker line element, and overdots stand for time derivatives.

Glanz (1995) has recently described the experimental work going on all over the world in order to determine the value for q_0, the present value of q.

Joshi (1993) stated that the observational data on the lower limits for the ages of stars and globular clusters, the redshift magnitude diagram, and the quasar data imply that

$$-4.4 < q_0 < 5.6 \tag{5}$$

Notice that the above range centers on the value $q_0 = 1$.

Earlier estimates, from two decades ago, given in the textbook by Adler *et al.* (1975), gave a narrower range of possible values, but also centered on the value $q_0 = 1$.

We now show that Mach's principle and general relativity theory lead to the value

$$q \sim 1 \tag{6}$$

The Whitrow–Randall relation can also be written as (Whitrow and Randall, 1951)

$$\frac{4\pi}{3} G\rho H^{-2} \sim 1 \tag{7}$$

where ρ is the energy density and H stands for Hubble's parameter,

$$H = \frac{\dot{R}}{R}$$

With a perfect fluid energy tensor, in the RW metric, Einstein's field equations read (Weinberg, 1972)

$$K\rho = 3H^2 - 3kR^{-2} - \Lambda \tag{8a}$$

$$Kp = -2\ddot{R}R^{-1} - H^2 - kR^{-2} + \Lambda \tag{8b}$$

where p is the cosmic pressure, k the tricurvature, Λ the cosmological constant, and

$$K = 8\pi G/c^2$$

On combining (4) and (8), we find

$$q = \frac{4\pi}{3} G\sigma H^{-2} \tag{9}$$

where we find

$$\sigma = \rho + 3p - 2\frac{\Lambda}{K} \tag{10}$$

Relations (9) and (10) do not depend on the value of k. As far as I know, this is the first time relation (9) has been published and claimed to be valid in this broad sense.

For the present universe, we may write

$$\sigma \cong \rho \tag{11}$$

because it is well understood that p and Λ are negligible.

On comparing (9) with (7), taking care of (10) and (11), we see that

$$q \sim 1 \tag{12}$$

We now go a step further and propose that

$$q = \frac{GM}{c^2R'} \tag{13}$$

Relation (13) is a novelty in cosmology, and we hope that it may lead to interesting developments.

In particular, making certain assumptions, we can equate R' to R. This would be at least valid for a closed universe ($k = +1$).

Berman (1994) recently remarked that the Planck universe obeys the Whitrow–Randall relation, so that Mach's principle must have been a property of the classical universe from its inception at 10^{-43} sec and it may be useful to consider a universe endowed with a constant unitary deceleration parameter ($q = 1$).

The details of such a universe have been worked out numerically by Berman and Gomide (1988; Berman, 1983). Though we need not repeat the existing calculation, we point out that this model has the following law of variation for Hubble's parameter:

$$H = DR^{-2} \tag{14}$$

where D is a constant whose value is, for a $k = +1$ universe, where pressure is temporarily zero,

$$D = c^2 H_0^{-1} \cong 5 \times 10^{28} \text{ cm}^2/\text{sec} \tag{15}$$

The other figures for such a universe are

$$R_0 = 2ct_0 \cong 1.7 \times 10^{28} \text{ cm} \tag{16}$$

$$\rho_0 \cong 1.1 \times 10^{-29} \text{ g cm}^{-3} \tag{17}$$

$$t_0 = \frac{H_0^{-1}}{2} \cong 9 \times 10^9 \text{ years} \tag{18}$$

$$M_0 \cong 10^{57} \text{ g}$$

If, instead, we had a flat universe, it would obey the following values for energy density, pressure, and scale factor:

$$\rho = \frac{3D^2}{K} R^{-4} \qquad (D = \text{const}) \tag{19}$$

$$p = \frac{\rho}{3} \tag{20}$$

$$R = (2Dt)^{1/2} \tag{21}$$

Observe that this is a radiation universe.

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